Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Subsidiary Level

Friday 6 June 2008 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y-coordinate of P is 8.

(a) Find, in terms of ln 2, the x-coordinate of P.

(2)

(b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found.

(4)

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2. $$f(x) = 5 \cos x + 12 \sin x.$$

Given that $f(x) = R \cos (x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(a) find the value of R and the value of α to 3 decimal places.

(4)

(b) Hence solve the equation

$$5 \cos x + 12 \sin x = 6$$

for $0 \leq x < 2\pi$.

(5)

(c) (i) Write down the maximum value of $5 \cos x + 12 \sin x$.

(1)

(ii) Find the smallest positive value of x for which this maximum value occurs.

(2)

3.

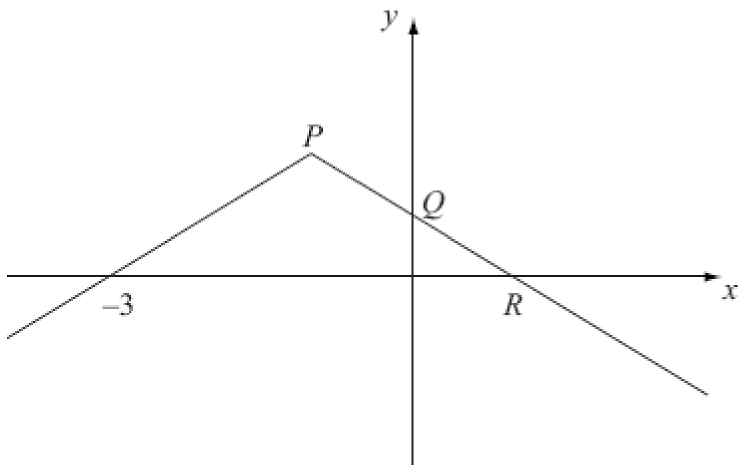

Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$,

The graph consists of two line segments that meet at the point P.

The graph cuts the y-axis at the point Q and the x-axis at the points (–3, 0) and R.

Sketch, on separate diagrams, the graphs of

(*a*) $y = |\mathrm{f}(x)|$,

(2)

(*b*) $y = \mathrm{f}(-x)$.

(2)

Given that $\mathrm{f}(x) = 2 - |x + 1|$,

(*c*) find the coordinates of the points P, Q and R,

(3)

(*d*) solve $\mathrm{f}(x) = \frac{1}{2}x$.

(5)

4. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \quad x > 3.$$

(*a*) Show that $\mathrm{f}(x) = \dfrac{1}{x+1}, \quad x > 3.$

(4)

(*b*) Find the range of f.

(2)

(*c*) Find $\mathrm{f}^{-1}(x)$. State the domain of this inverse function.

(3)

The function g is defined by

$$\mathrm{g}: x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(*d*) Solve $\mathrm{fg}(x) = \dfrac{1}{8}$.

(3)

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5. (*a*) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$.

(2)

(*b*) Solve, for $0 \leq \theta < 180°$, the equation

$$2\cot^2\theta - 9\operatorname{cosec}\theta = 3,$$

giving your answers to 1 decimal place.

(6)

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6. (*a*) Differentiate with respect to x,

(i) $e^{3x}(\sin x + 2\cos x)$,

(3)

(ii) $x^3 \ln(5x + 2)$.

(3)

Given that $y = \dfrac{3x^2 + 6x - 7}{(x+1)^2}$, $x \neq 1$,

(*b*) show that $\dfrac{dy}{dx} = \dfrac{20}{(x+1)^3}$.

(5)

(*c*) Hence find $\dfrac{d^2y}{dx^2}$ and the real values of x for which $\dfrac{d^2y}{dx^2} = -\dfrac{15}{4}$.

(3)

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7. $$f(x) = 3x^3 - 2x - 6.$$

(*a*) Show that f $(x) = 0$ has a root, α, between $x = 1.4$ and $x = 1.45$.

(2)

(*b*) Show that the equation f $(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

(*c*) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

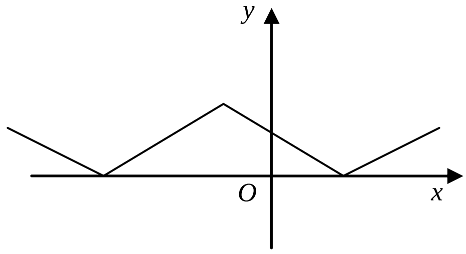
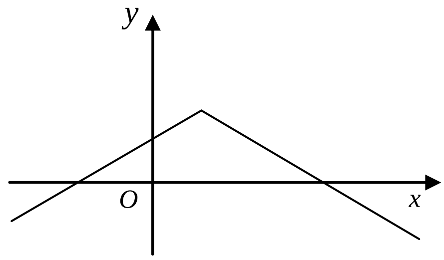
(*d*) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme		Marks
1. (*a*)	$e^{2x+1} = 2$		
	$2x + 1 = \ln 2$		M1
	$x = \frac{1}{2}(\ln 2 - 1)$		A1 (2)
(*b*)	$\frac{dy}{dx} = 8e^{2x+1}$		B1
	$x = \frac{1}{2}(\ln 2 - 1) \Rightarrow \frac{dy}{dx} = 16$		B1
	$y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right)$		M1
	$y = 16x + 16 - 8\ln 2$		A1 (4)
			(6 marks)
2. (*a*)	$R^2 = 5^2 + 12^2$		M1
	$R = 13$		A1
	$\tan\alpha = \frac{12}{5}$		M1
	$\alpha \approx 1.176$		A1 cao (4)
(*b*)	$\cos(x - \alpha) = \frac{6}{13}$		M1
	$x - \alpha = \arccos\frac{6}{13} = 1.091\ \ldots$		A1
	$x = 1.091\ \ldots + 1.176\ \ldots \approx 2.267\ldots$	awrt 2.3	A1
	$x - \alpha = -1.091\ \ldots$	accept … = 5.19 … for M	M1
	$x = -1.091\ \ldots + 1.176\ \ldots \approx 0.0849\ldots$	awrt 0.084 or 0.085	A1 (5)
(*c*)(i)	$R_{\max} = 13$ ft their R		B1 ft
(ii)	At the maximum, $\cos(x - \alpha) = 1$ *or* $x - \alpha = 0$		M1
	$x = \alpha = 1.176\ \ldots$	awrt 1.2, ft their α	A1ft (3)
			(12 marks)

Question Number	Scheme	Marks
3. (*a*)	 shape	B1
	Vertices correctly placed	B1 (2)
(*b*)	 shape	B1
	Vertex and intersections with axes correctly placed	B1 (2)
(*c*)	$P:(-1,2)$	B1
	$Q:(0,1)$	B1
	$R:(1,0)$	B1 (3)
(*d*)	$x>-1;\qquad 2-x-1=\frac{1}{2}x$	M1 A1
	Leading to $x=\frac{2}{3}$	A1
	$x<-1;\qquad 2+x+1=\frac{1}{2}x$	M1
	Leading to $x=-6$	A1 (5)
		(12 marks)

Question Number	Scheme	Marks
4. (*a*)	$x^2-2x-3=(x-3)(x+1)$	B1
	$f(x)=\dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}\quad\left(\textit{or}\ \dfrac{2(x-1)}{(x-3)(x+1)}-\dfrac{x+1}{(x-3)(x+1)}\right)$	M1 A1
	$=\dfrac{x-3}{(x-3)(x+1)}=\dfrac{1}{x+1}$ *****	A1 cso (4)
(*b*)	$\left(0,\frac{1}{4}\right)$ Accept $0<y<\frac{1}{4},\ 0<f(x)<\frac{1}{4}$ etc.	B1 B1 (2)
(*c*)	Let $y=f(x)$ $\quad y=\dfrac{1}{x+1}$	
	$x=\dfrac{1}{y+1}$	
	$yx+x=1$	
	$y=\dfrac{1-x}{x}$ or $\dfrac{1}{x}-1$	M1 A1
	$f^{-1}(x)=\dfrac{1-x}{x}$	
	Domain of f^{-1} is $\left(0,\frac{1}{4}\right)$	B1 ft (3)
(*d*)	$fg(x)=\dfrac{1}{2x^2-3+1}$	
	$\dfrac{1}{2x^2-2}=\dfrac{1}{8}$	M1
	$x^2=5$	A1
	$x=\pm\sqrt{5}$ both	A1 (3)
		(12 marks)

Question Number	Scheme	Marks
5. (*a*)	$\sin^2\theta+\cos^2\theta=1$	
	$\div\sin^2\theta \qquad \dfrac{\sin^2\theta}{\sin^2\theta}+\dfrac{\cos^2\theta}{\sin^2\theta}=\dfrac{1}{\sin^2\theta}$	M1
	$1+\cot^2\theta=\operatorname{cosec}^2\theta$ ∗	A1 cso (2)
(*b*)	$2\left(\operatorname{cosec}^2\theta-1\right)-9\operatorname{cosec}\theta=3$	M1
	$2\operatorname{cosec}^2\theta-9\operatorname{cosec}\theta-5=0$ *or* $5\sin^2\theta+9\sin\theta-2=0$	M1
	$(2\operatorname{cosec}\theta+1)(\operatorname{cosec}\theta-5)=0$ *or* $(5\sin\theta-1)(\sin\theta+2)=0$	M1
	$\operatorname{cosec}\theta=5$ *or* $\sin\theta=\dfrac{1}{5}$	A1
	$\theta=11.5°, 168.5°$	A1 A1 (6)
		(8 marks)
6. (*a*)(i)	$\dfrac{d}{dx}\left(e^{3x}(\sin x+2\cos x)\right)=3e^{3x}(\sin x+2\cos x)+e^{3x}(\cos x-2\sin x)$	M1 A1 A1 (3)
	$\left(=e^{3x}(\sin x+7\cos x)\right)$	
(ii)	$\dfrac{d}{dx}\left(x^3\ln(5x+2)\right)=3x^2\ln(5x+2)+\dfrac{5x^3}{5x+2}$	M1 A1 A1 (3)
(*b*)	$\dfrac{dy}{dx}=\dfrac{(x+1)^2(6x+6)-2(x+1)(3x^2+6x-7)}{(x+1)^4}$	M1 $\dfrac{\text{A1}}{\text{A1}}$
	$=\dfrac{(x+1)(6x^2+12x+6-6x^2-12x+14)}{(x+1)^4}$	M1
	$=\dfrac{20}{(x+1)^3}$ ∗	A1 cso (5)
(*c*)	$\dfrac{d^2y}{dx^2}=-\dfrac{60}{(x+1)^4}=-\dfrac{15}{4}$	M1
	$(x+1)^4=16$	M1
	$x=1,-3$ both	A1 (3)
		(14 marks)

Question Number	Scheme	Marks
7. (*a*)	$\mathrm{f}(1.4) = -0.568 \ \ldots < 0$	
	$\mathrm{f}(1.45) = 0.245 \ \ldots > 0$	M1
	Change of sign (and continuity) $\Rightarrow \ \alpha \in (1.4, 1.45)$	A1 (2)
(*b*)	$3x^3 = 2x + 6$	
	$x^3 = \dfrac{2x}{3} + 2$	
	$x^2 = \dfrac{2}{3} + \dfrac{2}{x}$	M1 A1
	$x = \sqrt{\left(\dfrac{2}{x} + \dfrac{2}{3}\right)}$ $\ast$	A1 cso (3)
(*c*)	$x_1 = 1.4371$	B1
	$x_2 = 1.4347$	B1
	$x_3 = 1.4355$	B1 (3)
(*d*)	Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval.	M1
	$\mathrm{f}(1.4345) = -0.01 \ldots$	
	$\mathrm{f}(1.4355) = 0.003 \ \ldots$	M1
	Change of sign (and continuity) $\Rightarrow \ \alpha \in (1.4345, 1.4355)$	
	$\Rightarrow \alpha = 1.435$, correct to 3 decimal places $\ast$ cso	A1 (3)
		(11 marks)